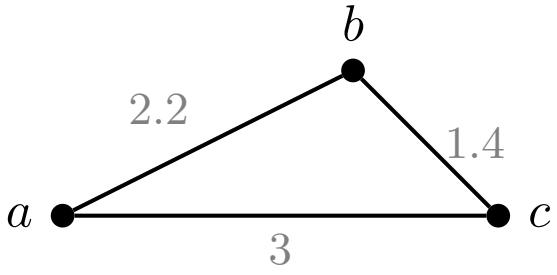
Tutorial 11

Trees and networks

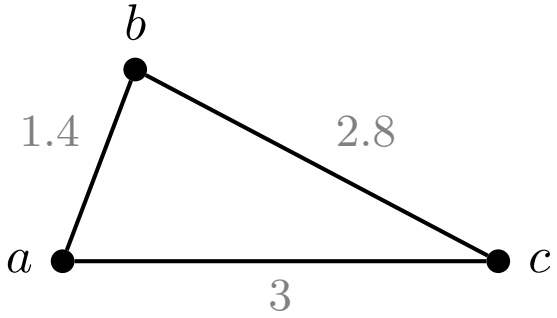
Question 1. For each of the following triangles

- Perform Torricelli's Construction to find the Fermat point. It is recommended that you use a protractor (or a compass) and a ruler.
- Compute the total length of the shortest network connecting the vertices of the triangle

1.



2.



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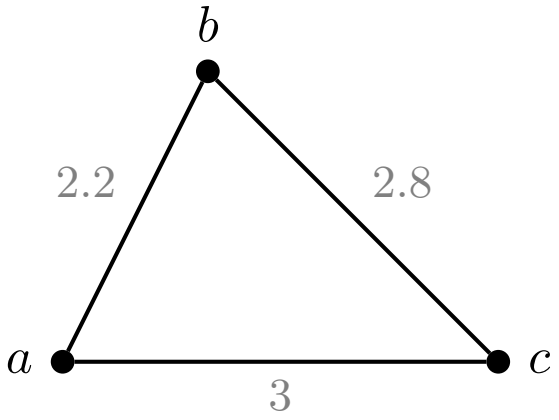

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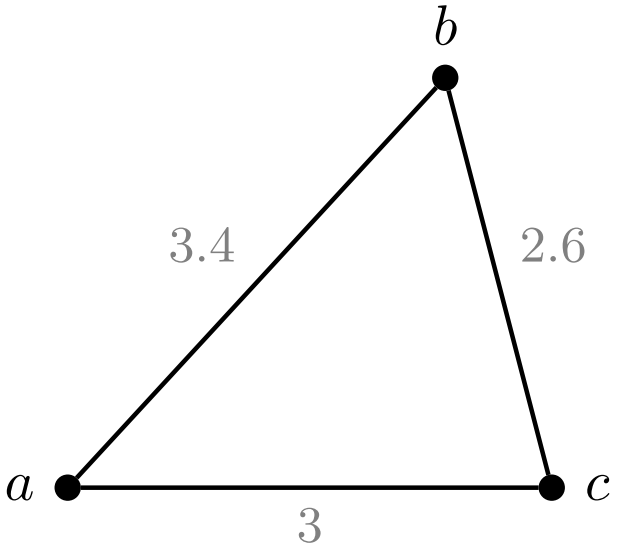


Solution.

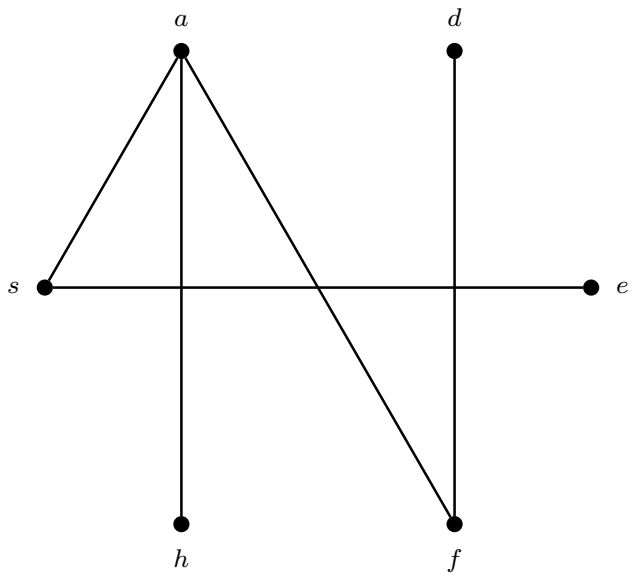
1. 3.633
2. 4.044
3. 4.625
4. 5.206

Question 2. Alice must visit clients in six cities next month and needs to minimize her driving mileage. The table below lists the distances between these cities. Use the mTSP Algorithm to find a good plan for her travels if she must start and end her trip in Dallas. Compute the total distance.

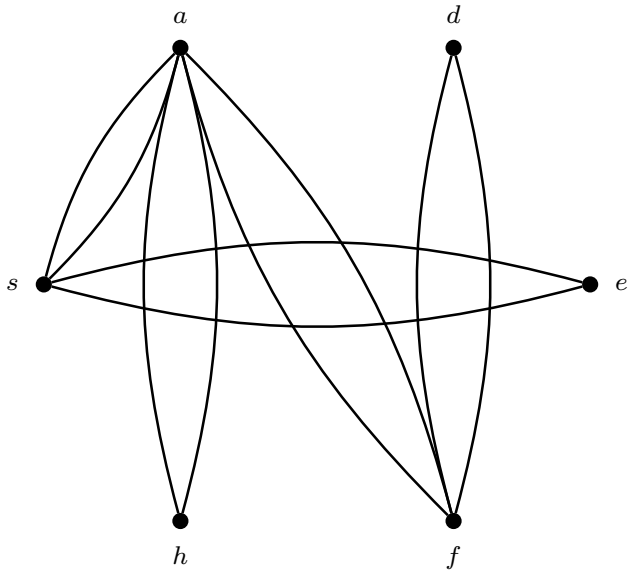
	Austin	Dallas	El Paso	Fort Worth	Houston	San Antonio
Austin	.	182	526	174	146	74
Dallas	182	.	568	31	225	253
El Paso	526	568	.	537	672	500
Fort Worth	174	31	537	.	237	241
Houston	146	225	672	237	.	189
San Antonio	74	253	500	241	189	.

Solution.

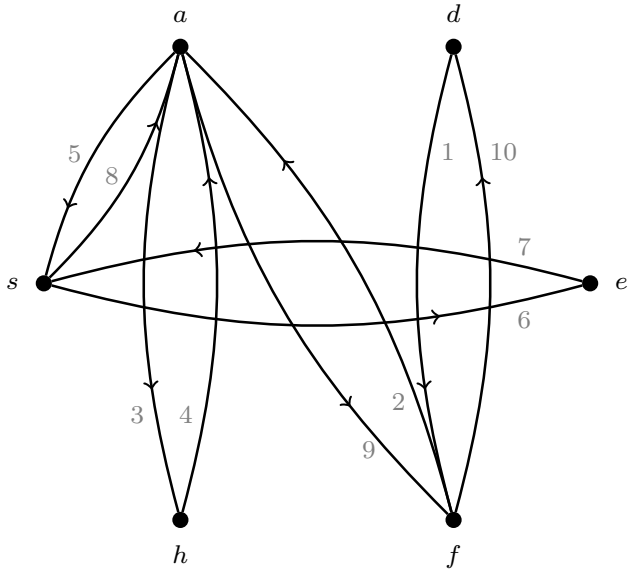
- Step 1. MST (by e.g. Prim's Algorithm)



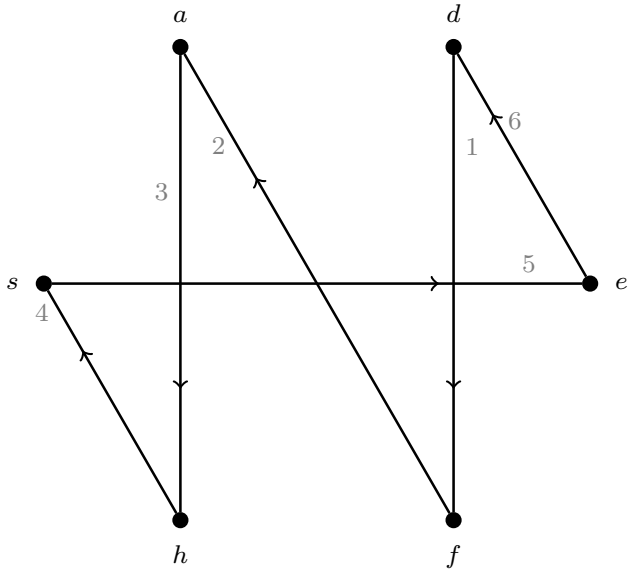
- Step 2. Duplicate edges



- Step 3. Find an Eulerian circuit



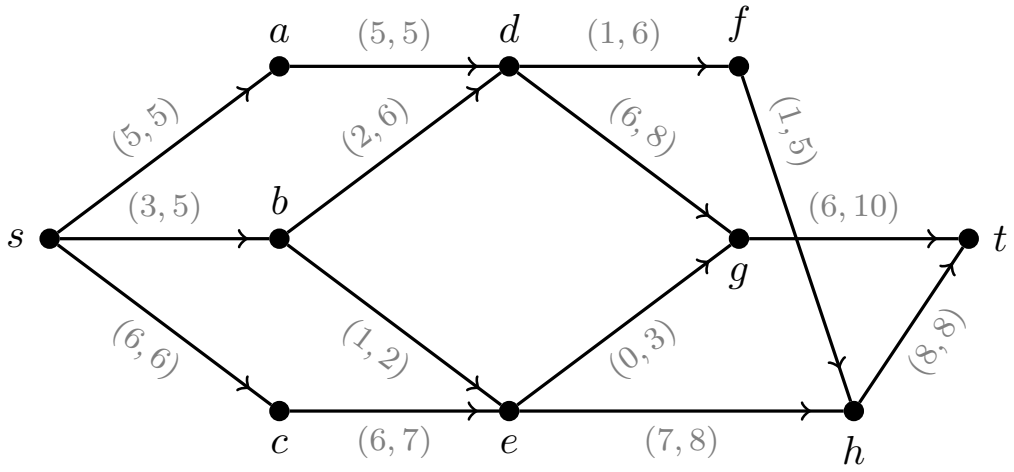
- Step 4. Hamiltonian cycle



- Step 5. Total weight 1608

Question 3. For each of the network below, use the Augmenting Flow Algorithm to maximize the flow and the Min-Cut Method to find a minimum cut.

1.



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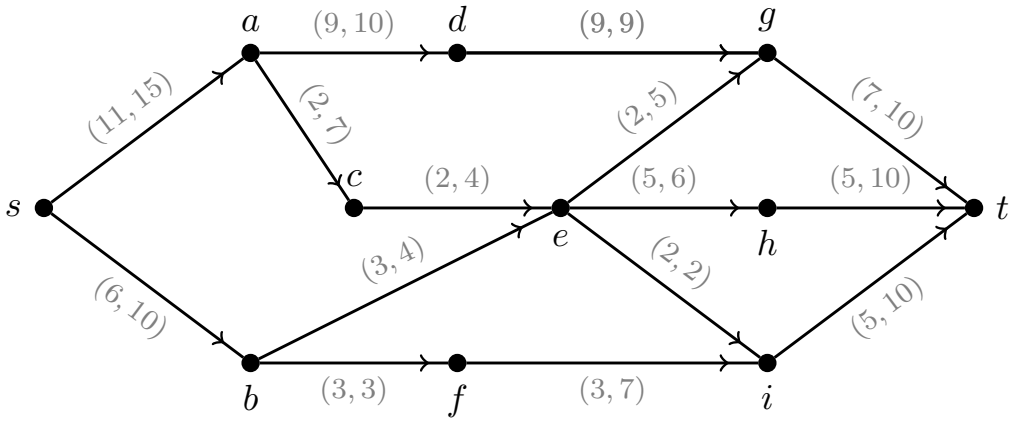

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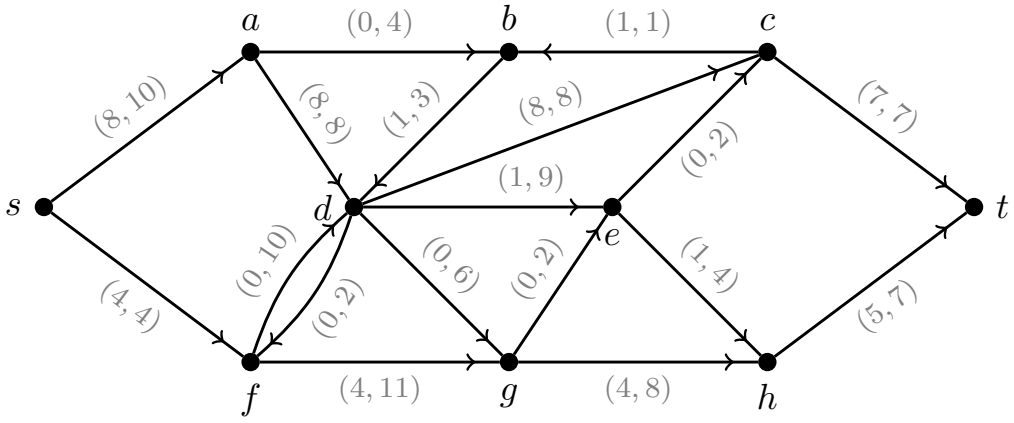

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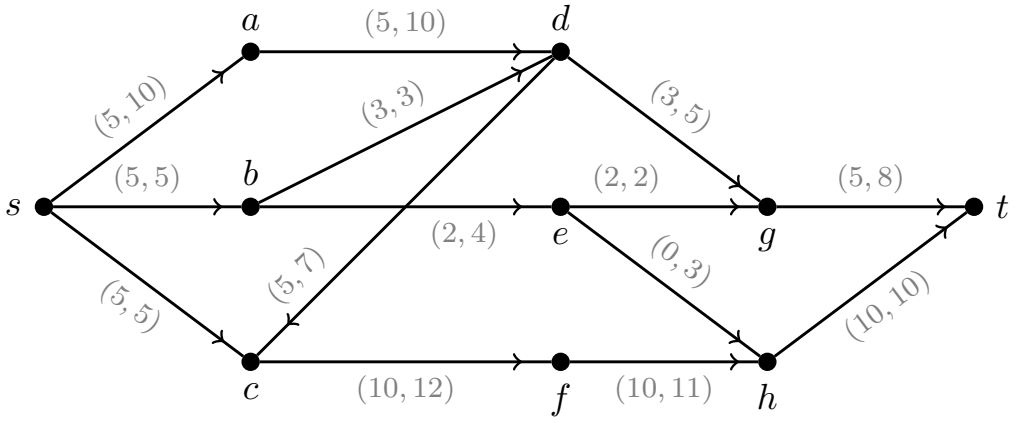
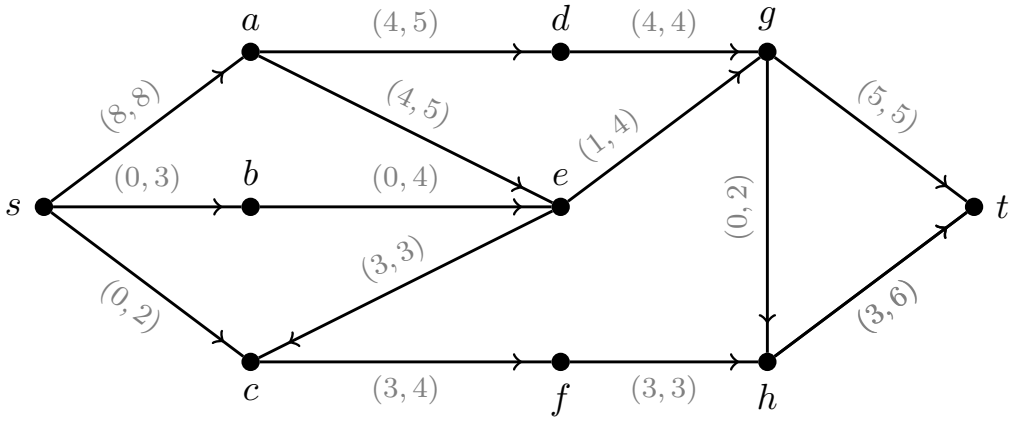
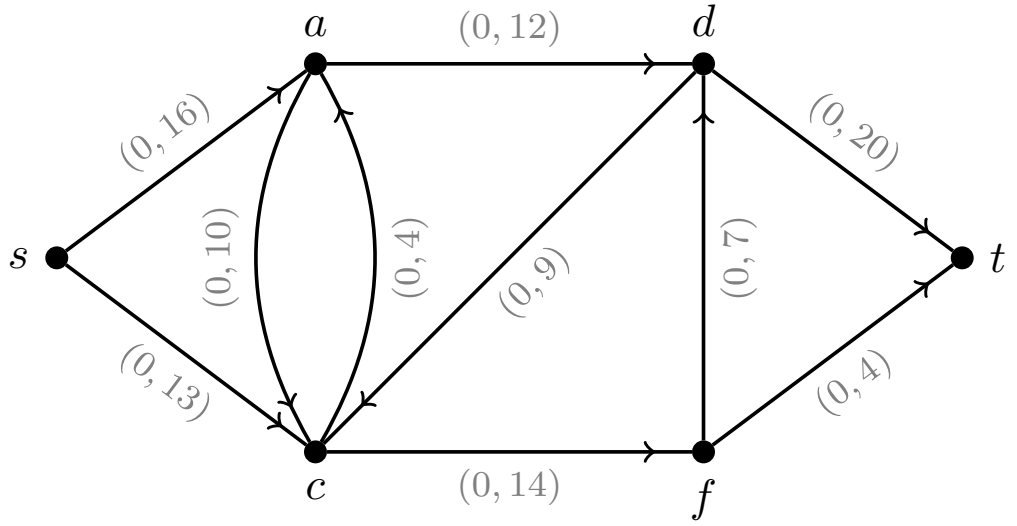

a
(5, 10)
d
(5, 10)
(3, 3)
(3, 5)
(5, 5)
b
(2, 2)
(5, 8)
s
t
(2, 4)
e
g
(0, 3)
(5, 7)
(5, 5)
(10, 10)
c
(10, 12)
f
(10, 11)
h

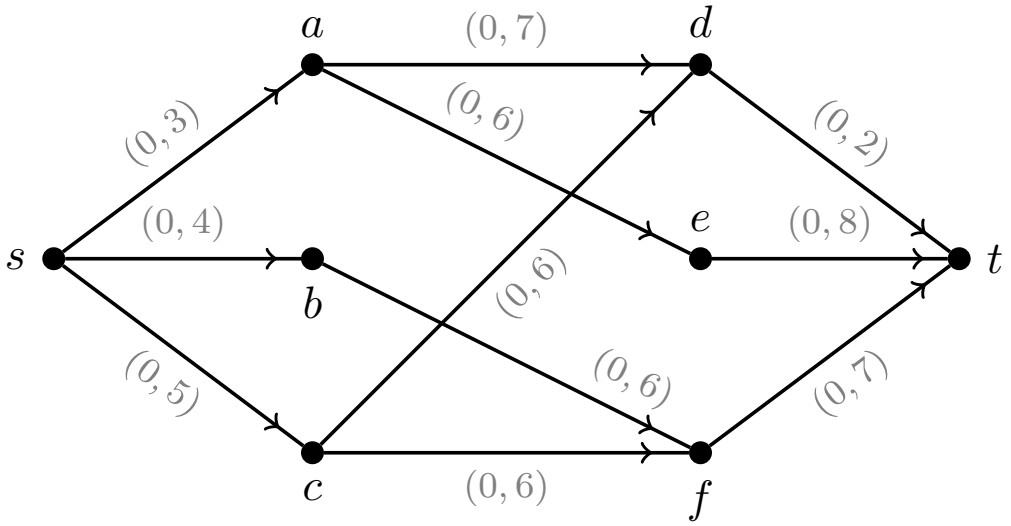
5.


a
(4, 5)
d
(4, 4)
g
(8, 8)
(4, 5)
(1, 4)
(5, 5)
(0, 3)
b
(0, 4)
e
s
t
(0, 2)
(3, 3)
(0, 2)
(3, 6)
c
(3, 4)
f
(3, 3)
h

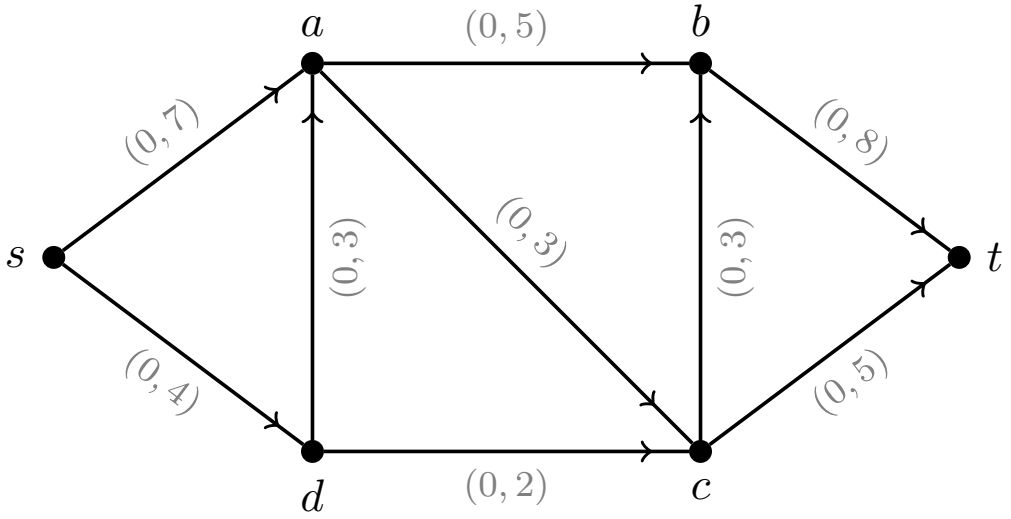
6.


a
(0, 12)
d
(0, 16)
(0, 20)
(0, 10)
(0, 4)
(0, 9)
(0, 7)
s
t
(0, 13)
(0, 4)
c
(0, 14)
f

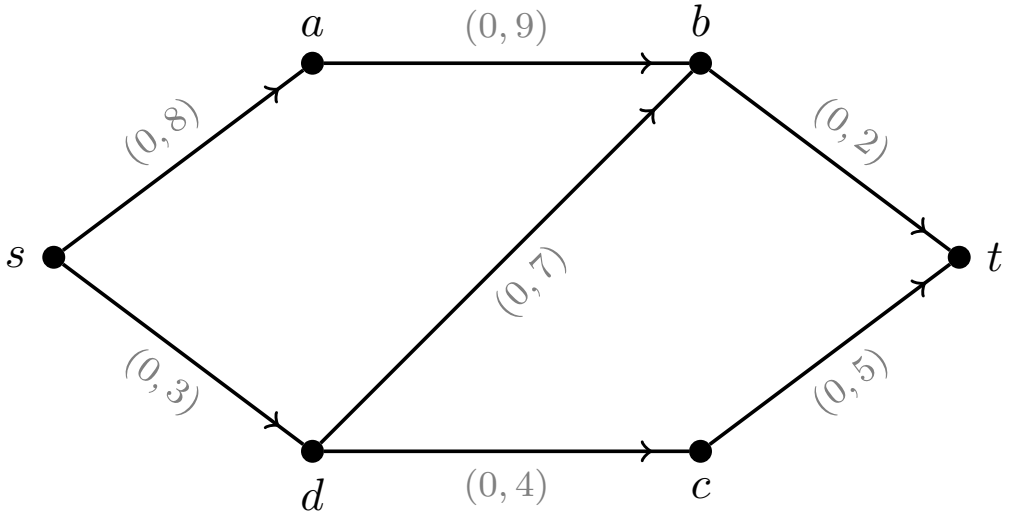
7.


a
(0, 7)
d
(0, 3)
(0, 6)
(0, 2)
(0, 4)
e
(0, 8)
s
t
b
(0, 6)
(0, 5)
(0, 6)
(0, 7)
c
(0, 6)
f

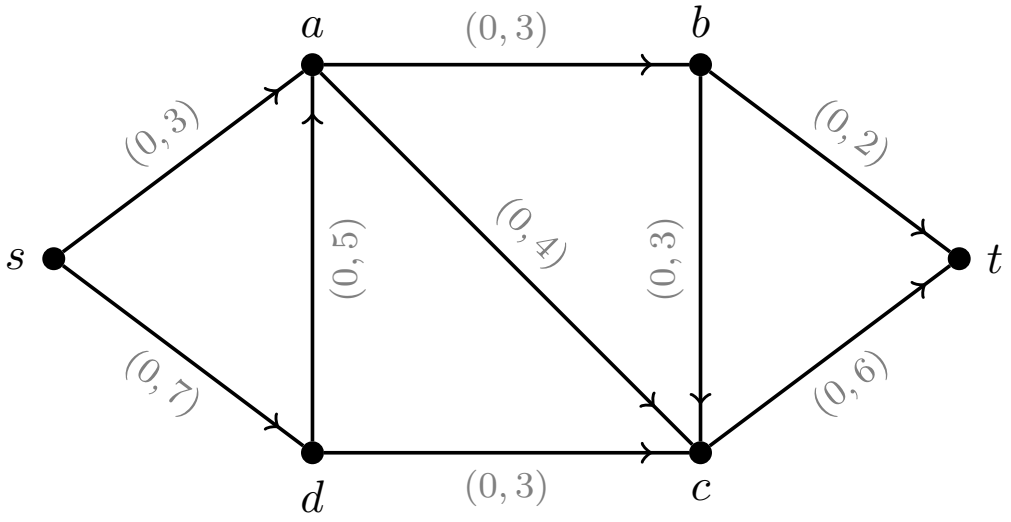
8.


a
(0, 5)
b
(0, 7)
(0, 8)
s
t
(0, 3)
(0, 3)
(0, 3)
(0, 4)
(0, 5)
d
(0, 2)
c

9.


a
(0, 9)
b
(0, 8)
(0, 2)
s
t
(0, 7)
(0, 3)
(0, 5)
d
(0, 4)
c

10.

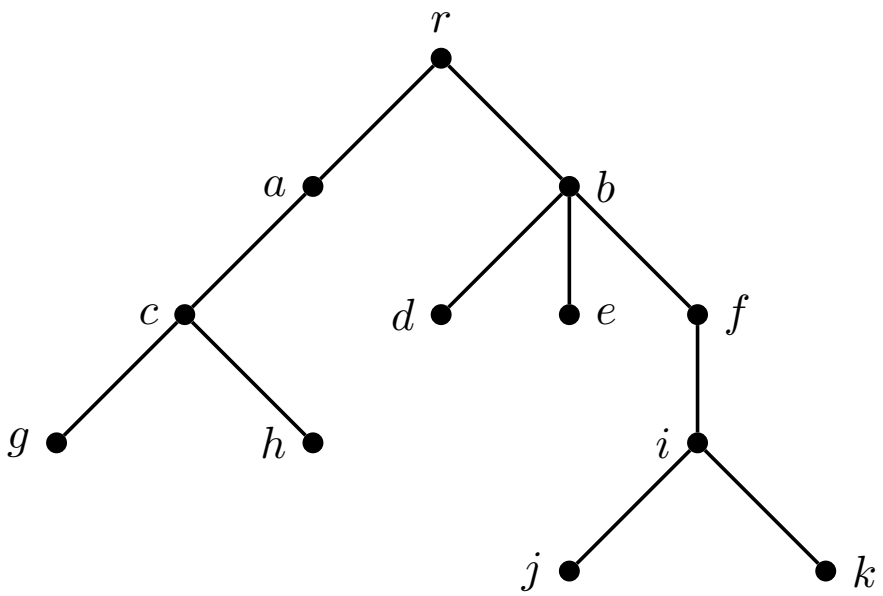


Solution.

1. Flow $= 16$, $P = \{s\}$
2. Flow $= 20$, $P = \{s, a, b, c, d\}$
3. Flow $= 14$, $P = \{s\}$
4. Flow $= 17$, $P = \{s, a, b, c, d, e, f\}$
5. Flow $= 10$, $P = \{s, a, b, c, d, e, f, g\}$
6. Flow $= 23$, $P = \{s, a, c, f\}$
7. Flow $= 12$, $P = \{s, d, c, b, f\}$
8. Flow $= 10$, $P = \{s, a, d\}$
9. Flow $= 5$, $P = \{s, a, b\}$
10. Flow $= 8$, $P = \{s, a, b, c, d\}$

Question 4. For the rooted tree T below, with root r, identify the following

1. Level of r, f, h, and k.
2. The height of T.
3. Parents of r, b, c, f, i.
4. Children of r, b, c, f, i.
5. Ancestors of r, a, d, h, k.
6. Descendants of r, a, d, h, k.
7. Siblings of a, f, h, i.



Solution.

1. $0; 2; 3; 4$
2. 4
3. none; $r; a; b; f$
4. $a, b; d, e, f; g, h; i; j, k$
5. none; $r; r, b; r, a, c; r, b, f, i$
6. $a, b, c, d, e, f, g, h, i, j, k; c, g, h;$ none; none; none
7. $b; d, e; g;$ none

Question 5. The main objective for a breadth-first search tree is to add as many neighbors of the root as possible in the first step. At each additional step, we are adding all available neighbors of the most recently added vertices. Its procedure can be described as follows

Breadth-First Search Tree

Input: Simple connected graph $G = (V, E)$ and a designated root vertex r
Steps:

1. Initialize the BFS tree $T = (V', E')$ with the root vertex r, i.e., $V' = \{r\}$, and mark r as visited.
2. Add all neighbors of r to V', and add the corresponding edges from r to each neighbor to E'. Mark all these neighbors as visited. Let this set of newly added vertices be the current level.
3. For each vertex v in the current level (in alphabetical order):
   - Add all unvisited neighbors x of v to V'.
   - Add the edge (v, x) to E'.
   - Mark each such neighbor x as visited.

   Let the collection of all such newly added vertices form the next level.
4. If T now includes all vertices of G, the process is complete. Otherwise, repeat step 3 using the next level as the current level.

Output: Breadth-first tree T

Complete each of the following on the two graphs shown below.

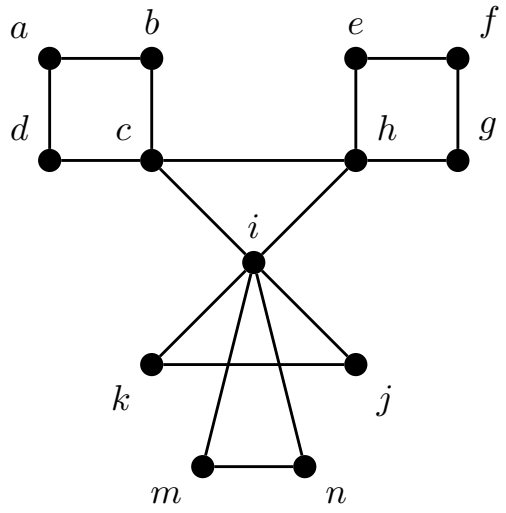
i) Find the breadth-first search tree with root a.

ii) Find the breadth-first search tree with root i.

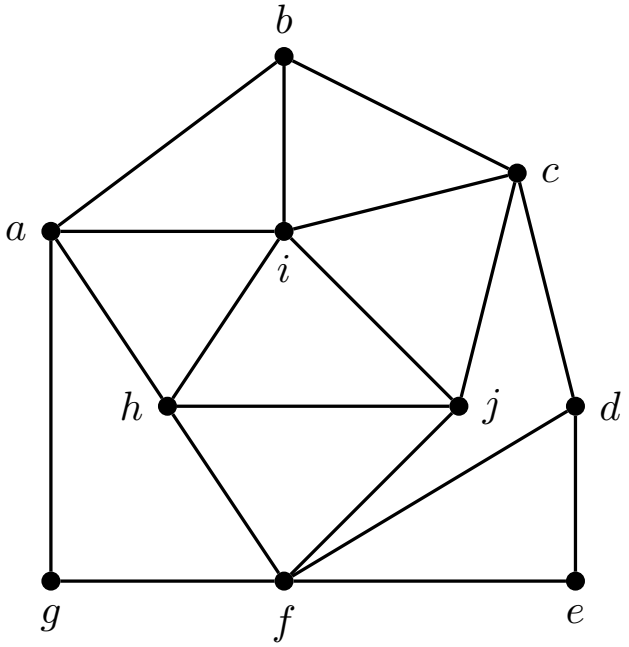
iii) Find the depth-first search tree with root a.

iv) Find the depth-first search tree with root i.

1.

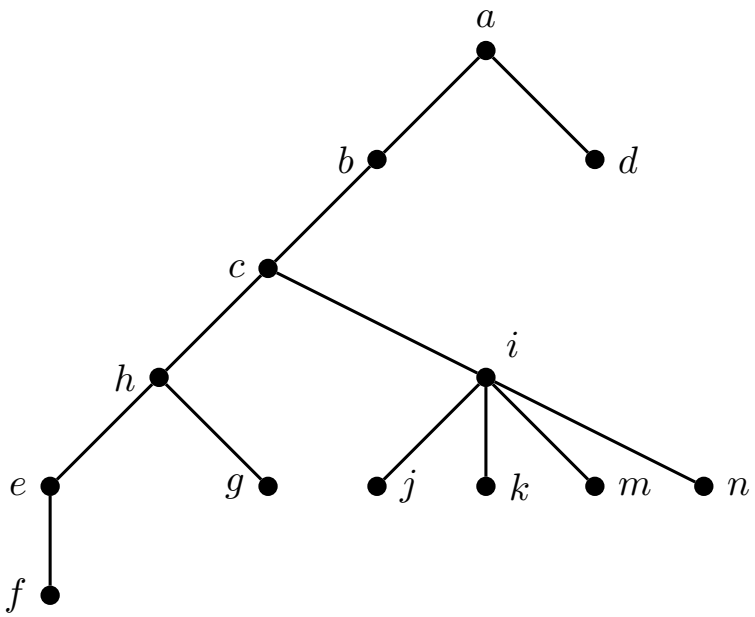


2.

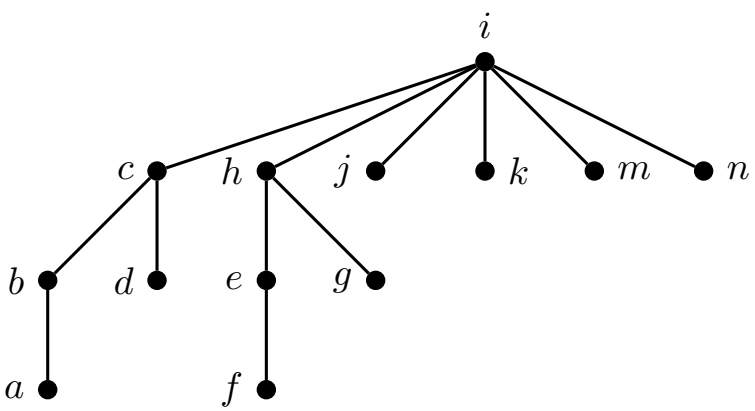


Solution.

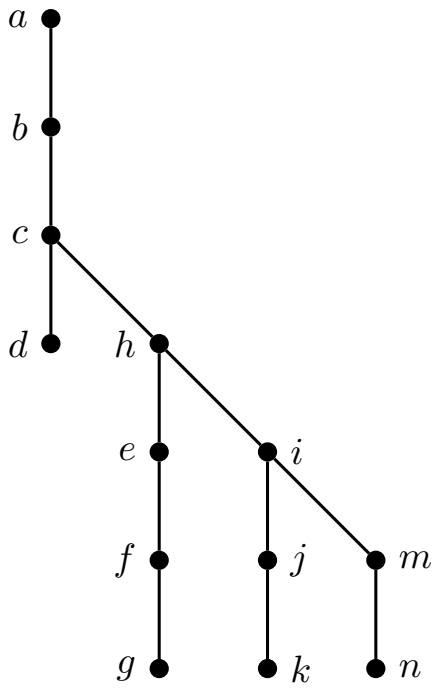
1. i) BFS tree with root a



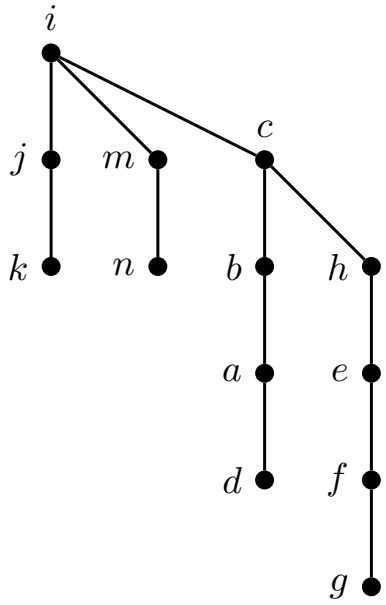
ii) BFS tree with root i



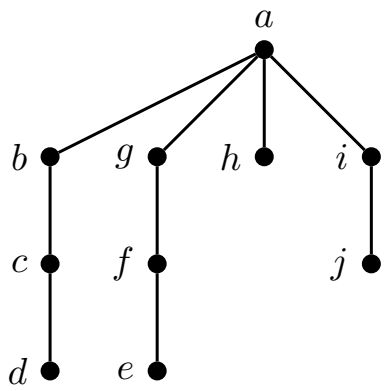
iii) DFS tree with root a



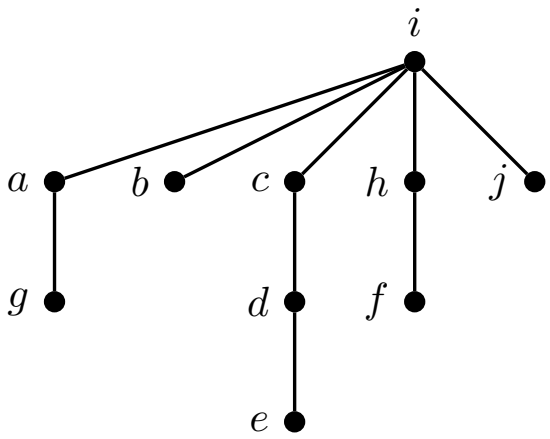
iv) DFS tree with root i



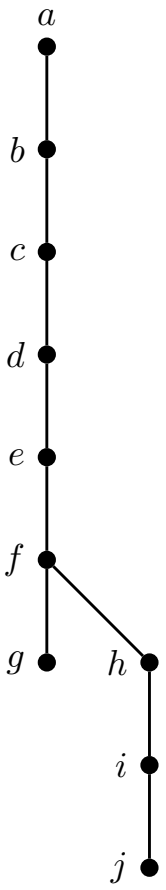
2. i) BFS tree with root a



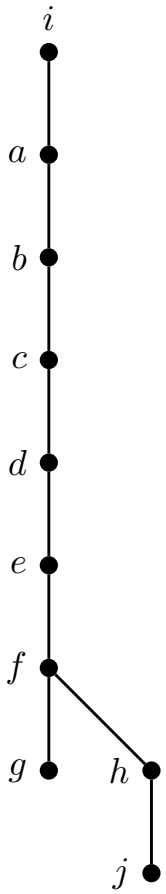
ii) BFS tree with root i



iii) DFS tree with root a



iv) DFS tree with root i


i
a
b
c
d
e
f
g
h
j