Tutorial 4

Determinants

Question 1. Compute the determinants of the following matrices using the standard determinant formula for 2×2 matrices and Sarrus' rule for 3×3 matrices.

1. $\begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$
2. $\begin{pmatrix} 5 & -3 \\ 2 & 0 \end{pmatrix}$
3. $\begin{pmatrix} 6 & -12 \\ -4 & 8 \end{pmatrix}$
4. $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$
5. $\begin{pmatrix} 2 & 0 & 5 \\ -4 & 1 & 7 \\ 0 & 3 & -3 \end{pmatrix}$
6. $\begin{pmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{pmatrix}$
7. $\begin{pmatrix} 5 & 0 & 0 \\ 3 & -2 & 0 \\ -1 & 8 & 4 \end{pmatrix}$
8. $\begin{pmatrix} -6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$
9. $\begin{pmatrix} 3 & 1 & -2 \\ -1 & 4 & 5 \\ 3 & 1 & -2 \end{pmatrix}$
10. $\begin{pmatrix} \alpha - 3 & 5 \\ -3 & \alpha - 2 \end{pmatrix}$
11. $\begin{pmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{pmatrix}$
12. $\begin{pmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{pmatrix}$
13. $\begin{pmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{pmatrix}$
14. $\begin{pmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{pmatrix}$
15. $\begin{pmatrix} \beta & -4 & 3 \\ 2 & 1 & \beta^2 \\ 4 & \beta - 1 & 2 \end{pmatrix}$

Question 2. Find all the minors and cofactors of the matrix A.

$$1.\ A = \begin{pmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{pmatrix}, \quad 2.\ A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{pmatrix}$$

Question 3. Let

$$A = \begin{pmatrix} 2 & 3 & -1 & 1 \\ -3 & 2 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 3 & -1 & 1 & 4 \end{pmatrix}$$

Find

1. Minor M_{32} and cofactor C_{32}
2. Minor M_{41} and cofactor C_{41}
3. Minor M_{44} and cofactor C_{44}
4. Minor M_{24} and cofactor C_{24}

Question 4. For each of the given matrices A, B, C, find the specified minors and cofactors.

$$A = \begin{pmatrix} 2 & 4 & 3 \\ 3 & -1 & 6 \\ 5 & -2 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 2 & -3 & 1 \\ 1 & 4 & 2 & -1 \\ 3 & -2 & 4 & 0 \\ 4 & -1 & 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} -3 & 3 & 0 & 5 \\ 2 & 1 & -1 & 4 \\ 6 & -3 & 4 & 0 \\ -1 & 5 & 1 & -2 \end{pmatrix}$$

1. Minors $M_{12}, M_{21}, M_{23}, M_{31}$ and cofactors $C_{12}, C_{22}, C_{32}, C_{21}$ for matrix A.
2. Minors $M_{12}, M_{42}, M_{24}, M_{31}, M_{34}$ and cofactors $C_{12}, C_{24}, C_{31}, C_{42}, C_{33}$ for matrix B.
3. Minors $M_{11}, M_{41}, M_{23}, M_{32}, M_{44}$ and cofactors $C_{12}, C_{24}, C_{31}, C_{42}, C_{33}$ for matrix C.

Question 5. Find the minors M_{11}, M_{31}, M_{23} and cofactors C_{12}, C_{32}, C_{22} for the following matrix

$$A = \begin{pmatrix} \alpha+1 & \alpha & \alpha-7 \\ \alpha-4 & \alpha+5 & \alpha-3 \\ \alpha-1 & \alpha & \alpha+2 \end{pmatrix}$$

Question 6. Compute the determinant of the given matrix. If the matrix is invertible, find its inverse using the formula for 2×2 matrices.

1. $\begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$
2. $\begin{pmatrix} 4 & 1 \\ 8 & 2 \end{pmatrix}$
3. $\begin{pmatrix} -5 & 7 \\ -7 & -2 \end{pmatrix}$
4. $\begin{pmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{pmatrix}$

Question 7. Find all values of λ for which $\det(A) = 0$.

1. $A = \begin{pmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{pmatrix}$
2. $A = \begin{pmatrix} \lambda-1 & 0 \\ 2 & \lambda+1 \end{pmatrix}$
3. $A = \begin{pmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{pmatrix}$
4. $A = \begin{pmatrix} \lambda-4 & 4 & 0 \\ -1 & \lambda & 0 \\ 0 & 0 & \lambda-5 \end{pmatrix}$
5. $A = \begin{pmatrix} \lambda & 2 \\ 5 & \lambda+3 \end{pmatrix}$
6. $A = \begin{pmatrix} 15 & \lambda-4 \\ \lambda+7 & -2 \end{pmatrix}$
7. $A = \begin{pmatrix} \lambda-3 & 5 & -19 \\ 0 & \lambda-1 & 6 \\ 0 & 0 & \lambda-2 \end{pmatrix}$

Question 8. Let

$$A = \begin{pmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{pmatrix}.$$

Compute the determinant of matrices A, B by a cofactor expansion along

1. the first row
2. the second row
3. the third row
4. the first column
5. the second column
6. the third column

Question 9. Compute the determinant of the following matrices by a cofactor expansion along a suitable row or column

1. $\begin{pmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{pmatrix}$
2. $\begin{pmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{pmatrix}$
3. $\begin{pmatrix} k+1 & k-1 & 7 \\ 2 & k-3 & 4 \\ 5 & k+1 & k \end{pmatrix}$
4. $\begin{pmatrix} 5 & 2 & 1 & 0 \\ -1 & 3 & 5 & 2 \\ 4 & 1 & 0 & 2 \\ 0 & 2 & 3 & 0 \end{pmatrix}$
5. $\begin{pmatrix} 0 & 5 & 4 & 0 \\ 4 & 1 & -2 & 7 \\ -1 & 0 & 3 & 0 \\ 0 & 2 & 1 & 5 \end{pmatrix}$
6. $\begin{pmatrix} 2 & 1 & 9 & 7 \\ 0 & -1 & 3 & 8 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 6 \end{pmatrix}$
7. $\begin{pmatrix} 3 & 3 & 0 & 5 \\ 2 & -2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{pmatrix}$
8. $\begin{pmatrix} 0 & 4 & 1 & 3 & -2 \\ 2 & 2 & 3 & -1 & 0 \\ 3 & 1 & 2 & -5 & 1 \\ 1 & 0 & -4 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 \end{pmatrix}$
9. $\begin{pmatrix} 0 & 2 & 3 & 4 & -1 \\ 0 & 1 & 0 & 0 & -3 \\ 1 & 4 & 2 & 0 & -3 \\ 1 & 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & -2 & 0 \end{pmatrix}$

Question 10. Evaluate the determinant of the given matrix by inspection.

1. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2. $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

3. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 4 & 3 & 0 \\ 1 & 2 & 3 & 8 \end{pmatrix}$

4. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

5. $\begin{pmatrix} 1 & 2 & 7 & -3 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 3 \end{pmatrix}$

6. $\begin{pmatrix} -3 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 40 & 10 & -1 & 0 \\ 100 & 200 & -23 & 3 \end{pmatrix}$

Question 11. For each matrix, show that the value of the determinant is independent of θ.

1. $\begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$

2. $\begin{pmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta - \cos\theta & \sin\theta + \cos\theta & 1 \end{pmatrix}$

Question 12. By inspection, what is the relationship between the following determinants?

$$d_1 = \begin{vmatrix} a & b & c \\ d & 1 & f \\ g & 0 & 1 \end{vmatrix}, \quad d_2 = \begin{vmatrix} a+\lambda & b & c \\ d & 1 & f \\ g & 0 & 1 \end{vmatrix}$$

Question 13. Show that

$$\det(A) = \frac{1}{2} \begin{vmatrix} \operatorname{tr}(A) & 1 \\ \operatorname{tr}(A^2) & \operatorname{tr}(A) \end{vmatrix}$$

for every matrix $A \in \mathcal{M}_{2\times 2}$.

Question 14. What can you say about $\det(A)$, where $A \in \mathcal{M}_{n\times n}$ has entries all equal to 1?

Question 15. What is the maximum number of zeros that a 3×3 matrix can have without having a zero determinant? Why?

Question 16. Explain why the determinant of a square matrix with integer entries must be an integer.

Question 17. Prove that the three points (x_1, y_1), (x_2, y_2), and (x_3, y_3) are collinear if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Question 18. Prove that the equation of the line through the distinct points (a_1, b_1) and (a_2, b_2) can be written as

$$\begin{vmatrix} x & y & 1 \\ a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \end{vmatrix} = 0$$

Question 19. Prove

Theorem 1 *A triangular matrix is invertible if and only if its diagonal entries are all nonzero.*

Question 20. Prove Theorem 3 using Theorems 1 and 2.

Theorem 2 *The inverse of an invertible lower triangular matrix is lower triangular, and the inverse of an invertible upper triangular matrix is upper triangular.*

Theorem 3 *If A is an invertible matrix, then*

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A).$$

Question 21. With the following examples, verify that $\det(A) = \det(A^\top)$

1. $A = \begin{pmatrix} -2 & 3 \\ 1 & 4 \end{pmatrix}$
2. $A = \begin{pmatrix} -6 & 1 \\ 2 & -2 \end{pmatrix}$
3. $A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{pmatrix}$
4. $A = \begin{pmatrix} 4 & 2 & -1 \\ 0 & 2 & -3 \\ -1 & 1 & 5 \end{pmatrix}$

Question 22. Each of the following matrices is obtained from the identity matrix by performing a single elementary row operation. Identify the specific operation performed and find the determinant of each matrix using the properties of determinants in relation to row operations.

1. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

3. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}$

4. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

5. $\begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

6. $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

7. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

8. $\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

9. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix}$

10. $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Question 23. Find the determinant of the following matrices using some combination of row operations, column operations, and cofactor expansion.

1. $\begin{pmatrix} 10 & 4 & 21 \\ 0 & -4 & 3 \\ -5 & -1 & -12 \end{pmatrix}$

2. $\begin{pmatrix} 18 & -9 & -14 \\ 6 & -3 & -5 \\ -3 & 1 & 2 \end{pmatrix}$

3. $\begin{pmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{pmatrix}$

4. $\begin{pmatrix} 3 & 6 & -9 \\ 0 & 0 & -2 \\ -2 & 1 & 5 \end{pmatrix}$

5. $\begin{pmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 5 & -2 & 2 \end{pmatrix}$

6. $\begin{pmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

7. $\begin{pmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{pmatrix}$

8. $\begin{pmatrix} -8 & 4 & -3 & 2 \\ 2 & 1 & -1 & -1 \\ -3 & -5 & 4 & 0 \\ 2 & -4 & 3 & -1 \end{pmatrix}$

9. $\begin{pmatrix} 1 & -1 & 5 & 1 \\ -2 & 1 & -7 & 1 \\ -3 & 2 & -12 & -2 \\ 2 & -1 & 9 & 1 \end{pmatrix}$

10. $\begin{pmatrix} 5 & 3 & -8 & 4 \\ \frac{15}{2} & \frac{1}{2} & -1 & -7 \\ -\frac{5}{2} & \frac{3}{2} & -4 & 1 \\ 10 & -3 & 8 & -8 \end{pmatrix}$

11. $\begin{pmatrix} 1 & 2 & -1 & 3 & 0 \\ 2 & 4 & -3 & 1 & -4 \\ 2 & 6 & 4 & 8 & -4 \\ -3 & -8 & -1 & 1 & 0 \\ 1 & 3 & 3 & 10 & 1 \end{pmatrix}$

12. $\begin{pmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & 1 & -4 \end{pmatrix}$

Question 24. Prove that if a square matrix has two proportional rows or columns, then its determinant is zero.

Question 25. Compute the following determinants, given that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$$

1. $\begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}$

2. $\begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix}$

3. $\begin{vmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{vmatrix}$

4. $\begin{vmatrix} a+d & b+e & c+f \\ -d & -e & -f \\ g & h & i \end{vmatrix}$

5. $\begin{vmatrix} a+g & b+h & c+i \\ d & e & f \\ g & h & i \end{vmatrix}$

6. $\begin{vmatrix} a & b & c \\ d & e & f \\ 2a & 2b & 2c \end{vmatrix}$

7. $\begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g+3a & h+3b & i+3c \end{vmatrix}$

8. $\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix}$

Question 26. Use row reduction to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

Question 27. Verify the following two formulas and make a conjecture about a general result of which these results are special cases

1. $\begin{vmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -a_{13}a_{22}a_{31}$

2. $\begin{vmatrix} 0 & 0 & 0 & a_{14} \\ 0 & 0 & a_{23} & a_{24} \\ 0 & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = a_{14}a_{23}a_{32}a_{41}$

Question 28. Confirm the following identities

1. $\begin{vmatrix} a_1 & b_1 & a_1+b_1+c_1 \\ a_2 & b_2 & a_2+b_2+c_2 \\ a_3 & b_3 & a_3+b_3+c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

2. $\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1-t^2)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

3. $\begin{vmatrix} a_1+b_1 & a_1-b_1 & c_1 \\ a_2+b_2 & a_2-b_2 & c_2 \\ a_3+b_3 & a_3-b_3 & c_3 \end{vmatrix} = -2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

4. $\begin{vmatrix} a_1 & b_1+ta_1 & c_1+rb_1+sa_1 \\ a_2 & b_2+ta_2 & c_2+rb_2+sa_2 \\ a_3 & b_3+ta_3 & c_3+rb_3+sa_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Question 29. For the following matrices, show that $\det(A) = 0$ without explicitly computing the determinant.

1. $A = \begin{pmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{pmatrix}$

2. $A = \begin{pmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & 1 & -4 \end{pmatrix}$

Question 30. It can be proved that if a square matrix M can be partitioned into *block triangular form* as

$$M = \begin{pmatrix} A & 0 \\ C & B \end{pmatrix} \quad \text{or} \quad M = \begin{pmatrix} A & C \\ 0 & B \end{pmatrix}$$

where A and B are square matrices, then

$$\det(M) = \det(A)\det(B).$$

Use this result to compute the determinants of the following matrices:

1. $M = \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 8 & 6 & -9 \\ 2 & 5 & 0 & 4 & 7 & 5 \\ -1 & 3 & 2 & 6 & 9 & -2 \\ \hline 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 8 & -4 \end{array}\right)$

2. $M = \left(\begin{array}{ccc|cc} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 2 \\ 2 & 0 & 0 & 0 & 1 \end{array}\right)$

Question 31. Given $A \in \mathcal{M}_{n\times n}$. Let B be the matrix that results when the rows of A are written in reverse order. Formulate a statement that describes the relationship between $\det(A)$ and $\det(B)$.

Question 32. Find the determinant of the following matrix.

$$\begin{pmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{pmatrix}$$

Question 33. Determine whether the given matrix is invertible by calculating its determinant. If the matrix is invertible, compute its inverse using the adjugate method.

1. $A = \begin{pmatrix} 5 & 6 \\ -3 & -4 \end{pmatrix}$

2. $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

3. $A = \begin{pmatrix} -12 & 7 & -27 \\ 4 & -1 & 2 \\ 3 & 2 & -8 \end{pmatrix}$

4. $A = \begin{pmatrix} 31 & -20 & 106 \\ -11 & 7 & -37 \\ -9 & 6 & -32 \end{pmatrix}$

5. $A = \begin{pmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{pmatrix}$

6. $A = \begin{pmatrix} 4 & 2 & 8 \\ -2 & 1 & -4 \\ 3 & 1 & 6 \end{pmatrix}$

7. $A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{pmatrix}$

8. $A = \begin{pmatrix} 1 & 0 & -1 \\ 9 & -1 & 4 \\ 8 & 9 & -1 \end{pmatrix}$

9. $A = \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{pmatrix}$

10. $A = \begin{pmatrix} -3 & 0 & 1 \\ 5 & 0 & 6 \\ 8 & 0 & 3 \end{pmatrix}$

11. $A = \begin{pmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{pmatrix}$

12. $A = \begin{pmatrix} \sqrt{2} & -\sqrt{7} & 0 \\ 3\sqrt{2} & -3\sqrt{7} & 0 \\ 5 & -9 & 0 \end{pmatrix}$

13. $A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -1 & 0 & -4 \end{pmatrix}$

Question 34. Evaluate the determinant of the given matrix by

(a) cofactor expansion

(b) row operations

1. $A = \begin{pmatrix} 2 & -2 & -2 & -2 \\ -2 & 2 & 3 & 0 \\ -2 & -2 & 2 & 0 \\ 1 & -1 & -3 & -1 \end{pmatrix}$

2. $A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -3 & -3 & -1 & -1 \\ -1 & -1 & -3 & 2 \\ -1 & -2 & 2 & 1 \end{pmatrix}$

3. $A = \begin{pmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 1 & 0 \end{pmatrix}$

4. $A = \begin{pmatrix} 1 & 0 & -1 & 0 & -1 \\ -1 & -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 \\ -1 & 1 & 1 & -1 & 0 \end{pmatrix}$

Question 35. Solve the following systems of equations using (if possible)

(a) the inverse of the coefficient matrix computed via the adjugate method

(b) Cramer's rule

1.

$$\begin{aligned} 7x_1 - 2x_2 &= 3 \\ 3x_1 + x_2 &= 5 \end{aligned}$$

2.

$$\begin{aligned} -9x - 4y &= 3 \\ -7x + 5y &= -10 \end{aligned}$$

3.

$$\begin{aligned} 2x + 3y &= 4 \\ 2x + 2y &= 4 \end{aligned}$$

4.

$$\begin{aligned} -10x - 7y &= -12 \\ 12x - 11y &= 5 \end{aligned}$$

5.

$$\begin{aligned} 5x - 5y &= 7 \\ 2x - 3y &= 6 \end{aligned}$$

6.

$$\begin{aligned} -x - 3y &= 4 \\ -8x + 4y &= 3 \end{aligned}$$

7.

$$\begin{aligned} 2x + 5y &= 4 \\ 4x + y &= 3 \end{aligned}$$

8.

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

9.

$$\begin{aligned} -2x + y - 4z &= -8 \\ -4y + z &= 3 \\ 4x - z &= -8 \end{aligned}$$

10.

$$\begin{aligned} 4x + 5y &= 2 \\ 11x + y + 2z &= 3 \\ x + 5y + 2z &= 1 \end{aligned}$$

11.

$$\begin{aligned} 2x + 3y + 2z &= -2 \\ -x - 3y - 8z &= -2 \\ -3x + 2y - 7z &= 2 \end{aligned}$$

12.

$$\begin{aligned} x - 4y + z &= 6 \\ 4x - y + 2z &= -1 \\ 2x + 2y - 3z &= -20 \end{aligned}$$

13.

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 4 \\ 2x_1 - x_2 &= -2 \\ 4x_1 - 3x_3 &= 0 \end{aligned}$$

14.

$$\begin{aligned} 3x_1 - x_2 + x_3 &= 4 \\ -x_1 + 7x_2 - 3x_3 &= 1 \\ 2x_1 + 6x_2 - x_3 &= 5 \end{aligned}$$

15.

$$\begin{aligned} 2x_1 + 2x_2 + 2x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned}$$

16.

$$\begin{aligned} -x_1 - 4x_2 + 2x_3 + x_4 &= -32 \\ 2x_1 - x_2 + 7x_3 + 9x_4 &= 14 \\ -x_1 + x_2 + 3x_3 + x_4 &= 11 \\ x_1 - 2x_2 + x_3 - 4x_4 &= -4 \end{aligned}$$

17.

$$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z - 2w &= 1 \\ 3x - 3w &= -3 \end{aligned}$$